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The puzzles of ground

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Abstract I outline and provide a solution to some paradoxes of ground.

Keywords Grounding · Puzzles of ground · Logic of ground · Worldly ground · Representational ground · Identification

1 Introduction

Here's a puzzle. Consider the fact that ⟨snow is white⟩ is true.[1] This fact is grounded by snow's being white. It is the fact that snow is white which explains why ⟨snow is white⟩ is true. This seems to be a general phenomenon. Truth is grounded in being: for any fact, A, A grounds ⟨A⟩ is true. Now consider the fact that something is white. If I have a ball of white snow, then this ball's being white grounds the fact that something is white. This seems to be another general phenomenon. Existential generalizations are grounded in their instances: any existential generalization, $\exists xFx$, is grounded in particular objects being F. But now we have a problem. For consider the fact that something is true. Call this fact S. Since truth is grounded in being, S grounds ⟨S⟩ is true. But ⟨S⟩ is true is an instance of S. So, since existential generalizations are grounded in their instances, ⟨S⟩ is true grounds S. So S grounds ⟨S⟩ is true and ⟨S⟩ is true grounds S. So we violate the asymmetry of ground and, assuming ground is transitive, we violate the irreflexivity of ground as well. But

[1] ⟨A⟩ refers to the proposition that A. So ⟨snow is white⟩ refers to the proposition that snow is white.

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grounding *is* asymmetric, transitive and irreflexive. So it seems we've landed ourselves in a contradiction. And so we have a puzzle.

This is a puzzle about the notion of ground. Ground is a distinctive type of non-causal explanation. Fine (2001) introduced this notion to contemporary metaphysics: it's now used widely across the field.[2] Some writers think the notion of ground is essential to articulating a layered conception of reality. Others think we should formulate key metaphysical theories in terms of ground. Still others think that the main aim of metaphysics should be to determine what grounds what.[3] We can tighten our grip on the notion with some examples. Consider the connection between sets and their members. The existence of these members is thought to explain the existence of sets. This explanatory connection is a connection of ground. The same is true of the connection between composite objects and their parts and between abstracta and concreta. The existence of parts is often thought to explain the existence of wholes. The existence of concreta is often thought to explain the existence of abstracta. Such connections are also expressed when we say that the physical explains the mental, the natural explains the normative and the determinate explains the determinable. These claims all express a distinctive kind of non-causal explanation: the notion of ground.

The puzzle is a puzzle for exactly this notion of ground. It highlights a tension between formal features of ground and claims about what grounds what. In this paper, I'll present a solution to this puzzle. The solution involves rejecting some plausible claims about what grounds what. But it's not good enough to baldly reject these claims. What we need to do is find something good to put in their place. This is key to solving the puzzle. My view is that by treating a particular notion of ground –weak ground– as fundamental we can generate that thing. We'll get to that view in Sect. 7. Beforehand we'll lay out the puzzle in more depth (Sects. 3–5). Then we'll see what others have said about it (Sect. 6). But let's start with some conceptual preliminaries.

2 Conceptual preliminaries

There are some distinctions between different notions of ground. These are important to this puzzle. Kit Fine makes these in his *Guide to Ground* (Fine 2012). The first important distinction is between full and partial ground. A full ground provides a completely satisfactory non-causal explanation of the thing grounded. We define partial ground in terms of full ground. Some fact, A, is a partial ground of another fact, B iff A, perhaps together with some other propositions, fully grounds B. In this case, A *helps* ground B. The above puzzle arises for both notions of ground. Later we'll see some related puzzles which only arise for the partial notion.

The second distinction is between factive and non-factive ground. Factive ground connects only truths. Non-factive ground also connects non-truths. We can say that

[2] He discovered this puzzle about ten years later (Fine 2010).

[3] See deRosset (2013a), Rosen (2010) and Schaffer (2009) for such claims.

A non-factively grounds the fact that $A \vee B$ even when A is false. But we would only say A factively grounds $A \vee B$ when A is true. There are a couple reasons to take non-factive ground to be more fundamental.[4] First, one can define factive (full) ground in terms of non-factive ground. The definition is: A factively grounds B iff A non-factively grounds B and A obtains. But nobody has been able to define factive grounds in terms of non-factive ground.[5] Second, rules governing factive ground are typically just less elegant versions of rules governing non-factive ground. With the non-factive notion we might have a rule that says: A non-factively grounds $A \vee B$. But with the factive notion we have: if A, then A factively grounds $A \vee B$. For these reasons, I'll focus on non-factive ground throughout. But it's often easy to generate parallel puzzles for factive ground.

The final important distinction is that between strict ground and weak ground. Strict ground is an irreflexive notion of ground. So, it's the notion of ground for which the puzzles arise. Weak ground is a reflexive notion. The puzzles don't arise for this notion of ground. We can understand strict ground as asymmetric weak ground. A strictly grounds B iff A weakly grounds B and B doesn't even help weakly ground A. There's a couple of ways to understand weak ground. We could understand it as disjunction. On this view, A weakly grounds B iff A is a disjunct of B. Or we could understand it as explanatory subsumption. On this view –very approximately– A weakly grounds B iff A explains everything B explains. We'll look at these in more depth in Sect. 7.1. For now let's just note that weak ground is sometimes considered the more fundamental notion. This is because treating it as such generates a cleaner logic.[6] Taking this thought seriously eventually generates the solution in this paper. But before seeing this let's explore our puzzle in more depth.

3 Formalizing the puzzle

It'll be helpful to formalize this puzzle. This makes clear what assumptions are in conflict. We begin by defining a formal language. Our language will have a vocabulary consisting of predicate letters (F, G, H...) and terms. The terms are variables, $(x, y, z\ldots)$ and constants. There will be two sorts of constants. The first sort of constant $(h, j, k\ldots)$ name objects. The second sort name well-formed formulas. For every wff in our language, A, there is a constant $\langle \mathrm{A} \rangle$. Interpret $\langle \mathrm{A} \rangle$ as the proposition that A. We also distinguish a special predicate, T. This is the truth-predicate. Finally we add parentheses, the logical constants ($\wedge$, $\vee$, $\neg$, $\exists$ and $\forall$), and operators for strict full ground ($<$) and strict partial ground ($\prec$).

We now describe the grammar of this language. To do this I'll use $A, B, C\ldots$ to represent arbitrary formulas and Δ, $\Gamma\ldots$ for arbitrary *lists* of formulas. A list is a

[4] Correia and Skiles (2017) are also inlined to take non-factive ground as basic.

[5] The only serious attempt I know of is (Fine manuscript). He isn't completely successful.

[6] This is the view in Fine (2009, 2012). It is also the view I take in Lovett (2019).

sequence of any finite number of basic formulas separated by ‘,’.[7] We define a well-formed formula as follows:

- If F is a predicate and t is a term, then Ft is a wff
- If A and B are wffs, then $(A \wedge B)$, $(A \vee B)$ and $(\neg A)$ are wffs
- If v is a variable and A is a wff, then $(\exists vA)$ and $(\forall vA)$ are wffs
- If A and B are wffs, then $(A \prec B)$ is a wff[8]
- If Δ is a list and B is a wff, then $(\Delta < B)$ is a wff

This completes our grammar. The proof theory will be a system of natural deduction.[9] Derivations take the form of a tree:

$$\frac{A_1; A_2 \ldots}{B}$$

Nodes in the tree are expressions of the grounding language. B is the root of the tree. The leaves of the tree are either inferred by valid rules of the form $\overline{B}$ or from hypotheses $A_1; A_2 \ldots$ via valid rules. You should interpret these trees in terms of ordinary validity: if the outermost leaves of a tree are true, then so must be the root. So they express general principles about how certain facts are grounded.

We can now see how our puzzle works in detail. It derives from the following rule schemas:

$$\text{T-I}\ \frac{}{A < T\langle A \rangle} \qquad \exists\text{-I}\ \frac{}{A < \exists v A^{[t/v]}}$$

$$\text{Asymmetry}(<)\ \frac{\text{A} < \text{B} \qquad \text{B} < \text{A}}{\bot}$$

Here $A^{[t/v]}$ is the result of uniformly substituting some term, t, in A for variable v and $\bot$ stands for any wff whatsoever. So T-I corresponds to the claim that truth is grounded in being. ∃-I corresponds to the claim that existential generalizations are grounded in their instances. Asymmetry(<) corresponds to the claim that strict full ground is asymmetric. This is because it says you can derive any formula (even contradictions) from asymmetric cases of ground.

Suppose these principles are all valid. Then it’s easy to generate the absurdity. Here goes:

$$\text{Asymmetry}(<)\ \frac{\text{T-I}\ \dfrac{}{\exists x Tx < T\langle \exists x Tx \rangle} \qquad \exists\text{-I}\ \dfrac{}{T\langle \exists x Tx \rangle < \exists x Tx}}{\bot}$$

This is just a simple formalization of the derivation which makes up our puzzle. So, what should we do? We have three options. We could reject asymmetry. We could reject T-I. Or we could reject ∃-I. The first option looks like a bad option. It

[7] We interpret the grammar of ‘,’ such that lists are invariant under both permutation and repetition: so we treat *A, B, C* …, for instance, as the same list as *C, B, A* … and *A* as the same list as *A, A, A* …

[8] This and the next clause mean I’m formulating ground as a sentential operator. This is common in the literature.

[9] As in Fine (2009, 2012).

seems very plausible that ground is asymmetric. This isn't a truth universally acknowledged. But it's probably a truth nonetheless.[10] Anyway we'll return to this option later (Sect. 6). But, for now, let's explore the other options. This will help us see what a good solution to this puzzle has to do. We'll begin with rejecting T-I.

4 Rejecting truth-introduction

Rejecting T-I would get us out of *this* puzzle. But it doesn't help with some closely related puzzles. Here's a plausible claim: knowledge too is partially grounded in being.[11] When S knows ⟨A⟩, then A partially grounds the fact that S knows ⟨A⟩. But now consider the fact that someone knows something. This isn't a recherché type of fact; it's just the claim that there is some knowledge. This fact is an existential generalization. So its instances must ground it. And one of its instances is Adam knows ⟨Someone knows something⟩. So the fact that Adam knows ⟨Someone knows something⟩ grounds the fact that someone knows something. But we said knowledge is at least partially grounded in being. So that someone knows something grounds that Adam knows ⟨Someone knows something⟩. So we have another violation of asymmetry.

Formally, we rely on slightly different principles to generate this problem. Expand our grammar to allow binary relations. Let K be the relation which expresses knowledge. The principles we rely on are:

$$\text{K-I}\ \frac{}{A \prec Ka\langle \mathrm{A} \rangle}$$

$$\text{Subsumption}(<\!/\!\prec)\ \frac{A, \Delta < B}{A \prec B} \qquad \text{Asymmetry}(\prec)\ \frac{\mathrm{A} \prec \mathrm{B} \qquad \mathrm{B} \prec \mathrm{A}}{\bot}$$

$$\text{Trans}(<\!/\!<)\ \frac{\mathrm{A} < \mathrm{B} \qquad \mathrm{B} < \mathrm{C}}{A < C}$$

Here K-I says that knowledge is partially grounded in being. Subsumption($<$/$\prec$) says that we can weaken full grounds into partial grounds. Asymmetry($\prec$) says that strict partial ground is asymmetric. Trans($<$/$<$) says strict full ground is transitive. If we endorse these principles, then rejecting T-I doesn't really get us out of the puzzle.[12]

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[10] Jenkins (2011), Rodriguez-Pereyra (2015), Thompson (2016) and Barnes (2018) do all reject this though.

[11] See Whitcomb (2012) for the original version of this puzzle. Rasmussen (2013) and Peels (2013) discuss it further.

[12] The derivation of an absurdity is:

$$\text{Asym}(<)\ \dfrac{\text{Sub}(<\!/\!\prec)\ \dfrac{\text{Trans}(<\!/\!<)\ \dfrac{\exists\text{-I}\ \dfrac{}{Ka\langle \exists x \exists y Kxy \rangle < \exists y Kay} \qquad \exists\text{-I}\ \dfrac{}{\exists y Kay < \exists x \exists y Kxy}}{Ka\langle \exists x \exists y Kxy \rangle < \exists y \exists y Kxy}}{Ka\langle \exists x \exists y Kxy \rangle \prec \exists x \exists y Kxy} \qquad \text{K-I}\ \dfrac{}{\exists x \exists y Kxy \prec Ka\langle \exists x \exists y Kxy \rangle}}{\bot}$$

We also need to reject K-I. But many instances of K-I look plausible. Suppose I look outside and see that it's raining. Then the fact that it is raining seems to partially ground the fact that I know ⟨it's raining⟩. So, if we reject K-I, we need some way of explaining the plausibility of these instances. Otherwise we incur a major loss of explanatory power. And that's a non-trivial cost.

Perhaps we might ignore this. We could reject K-I and T-I regardless. But even this won't solve these paradoxes. Stephan Krämer showed this a few years ago: we can generate puzzles without any predicational principles (Krämer 2013). To do this, we make use of (non-substitutional) quantification into sentence position. Let's expand our vocabulary to include sentential variables (P, Q, R...) and add the following two clauses to the grammar:

- All sentential variables are wff
- If A is a well-formed formula and V is a sentential variable, then $(\exists VA)$ and $(\forall VA)$ are wffs

Now consider the following principle:

$$\exists\text{-I}^h \ \frac{}{A < \exists V A^{[B/V]}}$$

where $A^{[B/V]}$ is the result of uniformly substituting any wff, B, in A for the variable V. This extends ∃-I to the case of sentential quantification. It seems disunified not to have this higher-order principle given we have its first-order counterpart. Such disunity would seem to merit an explanation. But it's not obvious what such an explanation would be.[13] So endorsing ∃-I seems to push us into endorsing its higher-order counterpart. But the higher-order counterpart generates a very simple violation of irreflexivity. Consider the following instance of $\exists$-I^h:

$$\exists\text{-I}^h \ \frac{}{\exists PP < \exists PP}$$

This is a violation of the irreflexivity of ground. So we can generate puzzles without predicational rules like T-I. The strategy of rejecting such rules is insufficiently general: it does not solve all the puzzles.[14]

[13] Some people, for example (Korbmacher 2018a), formalize ground as a predicate of sentences. Currently, such systems only deal with sentences which express first-order claims. They have not yet been extended to sentences which express higher-order claims. So, such an extension might not yield a higher-order rule which generates a Krämer-like paradox. If so, taking ground to be a predicate would provide an interesting way to avoid this paradox. But how promising this option is is as yet unknown: it depends on how we should extend systems like the one in Korbmacher (2018a).

[14] This extension of the system also puts us in a position to deal with a worry about the puzzle concerning knowledge. The worry is that the plausibility of K-I rests entirely on that of T-I. This would mean the knowledge puzzle added nothing further to the puzzle involving truth. One might think this for the following reason: suppose one endorses the claim that, generally, T⟨A⟩ strictly partially grounds Ka⟨A⟩. The instances of K-I follow from this, T-I and the transitivity of partial ground. So, one might think that these rules provide the basis for the intuition that these instances are plausible. So, if we jettison T-I, then we have no reason to endorse K-I. I'm unsure whether this is a good worry. That's because I'm unsure what the basis of my intuitions are. But, as I said, the extension of the system allows us to deal with it. To do so, we introduce a relation, K^h, which can hold between terms and wffs. We interpret $K^h tP$ as t knows

5 Rejecting existential-introduction

Here's a different strategy: reject the existential generalization rule. But parallel problems arise here. Namely, rejecting this doesn't get us out of some similar puzzles. For instance, it fails to extricate us from a puzzle involving universal generalization. It seems plausible that universal generalizations are partially grounded in their instances. So, the fact that all water is wet is partially grounded by the fact that *this* water is wet. But now consider any universal generalization about truth. For instance, consider the (false) claim that everything is true. Call this claim S*. An instance of this claim is ⟨everything is true⟩ is true. Hence, ⟨everything is true⟩ is true partially (non-factively) grounds S*. But if truth is grounded in being, then S* also partially (non-factively) grounds ⟨everything is true⟩ is true. So we have violation of asymmetry all the same.[15]

So one might reject both quantificational rules. But there are a few puzzles which don't arise from these quantification rules. They arise when we consider what grounds the truth of *sentences*. Consider any sentence, 'S', such that 'S' expresses A. It seems plausible that A strictly partially grounds the fact that 'S' is true. Attributions of truth to sentences are grounded, in part, by the facts those sentences express. Some puzzles now emerge. Here's the first: suppose there is some sentence, 'D', such that 'D' expresses ('D' is true or 'D' is not true). It follows that ('D' is true or 'D' is not true) strictly partially grounds 'D' is true. And suppose that disjunctions are grounded in their disjuncts. It follows that 'D' is true strictly grounds ('D' is true or 'D' is not true). And this is a violation of asymmetry. Here's the second puzzle. Suppose there is some sentence, C, such that C expresses ('C' is true and 'C' is a sentence). It follows that ('C' is true and 'C' is a sentence) partially grounds 'C' is true. But now suppose that conjunctions are partially strictly grounded in their conjunctions. It follows that 'C' is true partially grounds ('C' is true and 'C' is a sentence). We've got a second asymmetry violation. And there's also a third puzzle. Suppose there's a sentence, T, such that T expresses 'T' is true. It follows that 'T' is true grounds 'T' is true. This violates irreflexivity. Call this final puzzle the puzzle of the truthteller.[16] These three puzzles provide violations of asymmetry quite independently of the quantificational rules.

These puzzles are a bit different from our previous puzzles. A major difference is that they rely on existence assumptions. The puzzle of the truthteller, for instance, requires we assume that there's some sentence, 'T', which expresses 'T' is true. The evidence for these assumptions is straightforward. It seems like we can give examples of such sentences. 'This sentence is true' is such an example. This

Footnote 14 continued

that P. We then formulate a new version of the knowledge rule. This is: $\overline{A \prec K^h aA}$. This, together with ∃-I^h, generates absurdity. The proof parallels that in n.12: just remove all the propositional brackets, replace ∃-I for ∃-I^h, and replace K-I for this new rule. There's no obvious reason for thinking the plausibility of this rule is based on T-I. So we've dealt with the worry.

15 This exact puzzle doesn't arise on the factive notion. But Fine (2010, 102) outlines a related one which does.

16 Fine (2010) and Rodriguez-Pereyra (2015) both remark on this.

sentence seems to say of itself that it's true. And, as is well known, there are also sentences which seem to contingently exemplify these existence assumptions. Suppose 'The sentence on the blackboard in room 202 is true' is the only sentence on the blackboard in room 202. Then this sentence seems to say of itself that it is true. So, there's strong intuitive evidence to accept these existence assumptions. Now, such evidence isn't always taken to be decisive. Some have thought that some sentences, despite appearances, don't express anything at all. They've thought this about the Liar sentence.[17] The Liar is the sentence 'this sentence is false.' Those who have thought this have been motivated by the fact that, if we take the Liar to express something, we can derive a contradiction from classical logic and the truth-schema alone.[18] But, importantly, this fact doesn't motivate denying that 'this sentence is true' expresses something. So it doesn't motivate denying that it expresses its own truth. The truthteller is not paradoxical in the way the Liar sentence is. So these approaches to the Liar paradox provide little reason to doubt our existence assumptions.

So the existence assumptions seem to me plausible. But let's clarify a couple of things about them. First, they're assumptions about the existence of sentences. They're not assumptions about the existence of propositions. I've claimed it's plausible that some sentences self-attribute truth. But it's not at all obvious that there are propositions which self-attribute truth. So, these puzzles do not arise very forcefully in the case of propositional truth. Second, they are just assumptions about *existence*. They're not assumptions about truth values. I've claimed it's plausible that sentences which self-attribute truth exist. But it's not at all obvious whether such sentences are true.[19] I needn't assume they're true, because we're working with the non-factive notion of ground. Non-factive ground can hold between non-truths. So these puzzles also do not arise very forcefully in the case of factive ground.[20] These puzzles concern the interaction of non-factive ground and attributions of truth to sentences.

For all that, these puzzles –especially the puzzle of the truthteller– seem to me quite important. They make it difficult to respond to our puzzles by rejecting the logical rules. The first two puzzles rely on rules beyond the quantificational. And the puzzle of the truthteller relies on no such rules. So we can't evade these particular puzzles by denying such rules. This closes an important way of solving these puzzles of ground.

With this in mind, it'll be useful to formally state what principles these puzzles rely on. To do so we expand our language a little. For every wff of the language, A, add a constant 'A' to our vocabulary. Interpret 'A' as the sentence which expresses

[17] This is part of contextualist approaches to the liar paradox. See, for example, Parsons (1974, 385–390) and Glanzberg (2004, 33–34).

[18] The truth schema says: $A \leftrightarrow T\langle A\rangle$.

[19] Kripke (1975) famously denies that they are.

[20] However, see Rodriguez-Pereyra (2015, 525–528) for a defence of factive versions of the puzzle of the truthteller.

A. We also distinguish a special connective, 'expresses.' This can hold between wff and constants. These puzzles rely on the following principles:

$$\text{T-I}^{s}\ \frac{\text{`A' expresses A}}{\text{A} \prec \text{T `A'}}$$

$$\vee\text{-I}_1\ \frac{}{A < A \vee B} \qquad \vee\text{-I}_2\ \frac{}{B < A \vee B}$$

$$\wedge\text{-I}\ \frac{}{A, B < A \wedge B}$$

$$\text{TT}_1\ \frac{\exists x(x \text{ expresses } \text{T}x \vee \neg\text{T}x)}{} \qquad \text{TT}_2\ \frac{\exists x(x \text{ expresses } \text{T}x \wedge \text{F}x)}{}$$

$$\text{TT}_3\ \frac{\exists x(x \text{ expresses } Tx)}{}$$

$$\exists\text{-Elim}\ \frac{\exists x A}{\text{A}^{[v/t]}}$$

Here the TT rules encapsulate the existence assumptions, whilst T-I^{s} captures the claim about how attributions of truth to sentences are grounded. These allow us to generate the three puzzles above.

So let's take stock. What's the issue with the two strategies we've looked at? The obvious issue is that neither strategy, on it's own, solves all the puzzles. Each strategy lacks breadth. Each leaves something out. Rejecting the logical rules does not deal with all the puzzles. It leaves out the puzzle of the truthteller. Rejecting the predicational rules also doesn't deal with all the puzzles. It leaves out Krämer's higher-order puzzle. So these strategies on their own are inadequate. But why not embark on these strategies in concert? Why not reject all these rules? In a sense this is what I'll end up suggesting. But I think baldly denying these principles is unsatisfactory. This is because many instances of these principles *do* seem true. This is clearest with the logical rules. The fact that my jacket is red does strictly ground the fact that something is red. The fact that my shoes are self-identical does strictly (partially) ground the fact that everything is self-identical. But suppose we baldly deny these rules. Then we're left with no explanation of when the true instances are true and when the false ones are false. This seems to me unsatisfactory. It degrades our explanatory position. What we need, it seems to me, is some way of systematically restricting these principles. I'll provide that in Sect. 7.

6 What have other people said?

Before doing that, let's look at a few extant solutions to this puzzle. I don't think these solutions are fully satisfactory. In some cases, this is because they don't deal with the full breadth of the puzzle. In other cases, it's because they lack explanatory depth. But I don't aim to say anything decisive against these solutions. All I want to do is point out a few problems with them.

Let's start with Fine. Fine (2010) relates these puzzles to the semantic paradoxes. Actually, that's not quite true. Fine relates some different puzzles to the semantic paradoxes. The puzzles he relates are puzzles for the factive notion of ground. He considers various solutions to these puzzles of factive ground. Each solution parallels an approach Kripke considers for the semantic paradoxes. Fine himself doesn't endorse any of these solutions. But we should see if they work anyway. I myself am sceptical.

Most of the solutions Fine considers involve abandoning what he calls "logical principles" (Fine 2010, 107–11). These are principles such as everything is true or false, and something is true. Abandoning these principles gets rid of some versions of the puzzle. For instance, if we deny $\exists xTx$ then we can't apply the factive version of our truth introduction rule. So the factive version of our first puzzle disappears.[21] So these solutions help with the puzzles Fine raises. But none of these solutions help with the puzzles for non-factive ground. The puzzles for non-factive ground do not depend on such principles. So these solutions are not sufficiently general.

One could just eschew non-factive ground. But even then these solutions seem unattractive. This is because of the breadth of the puzzles. A thoroughgoing version of any of these solutions must fail to accept that anything is true, that anybody knows anything and that $\exists PP$. This does not seem to me a good solution to the puzzles. It seems to me obvious that something is true and that some knowledge exists. And $\exists PP$ arises from existential instantiation on any tautology. So even for the factive notion this solution seems to me untenable. So, generally, rejecting these logical principles seems unpromising.

Fine does consider a solution which one can extend to non-factive ground. He calls this the supervaluational approach. This parallels the supervaluational treatment of logically complex sentences in a Kripke construction. Explaining this takes some doing. So I refer the interested reader to Fine (2010, 111–12). For our purposes, it's enough to note two things. First, this solution entails no logical truths have a ground at all. This is a major cost. It seems to me quite implausible that nothing ever grounds any logical truth. Second, this solution doesn't solve all our puzzles. It fails to resolve the puzzle of the truthteller. This is because this puzzle does not rely on any principles governing logically complex sentences. So the supervaluational approach also seems insufficiently general.

Let's move onto some different solutions. Korbmacher (2018b) explores a typed solution to these puzzles. He does this by formalizing ground are a predicate instead of an operator. But let's see how such a solution would apply to our puzzles. When it comes to truth, we can throw out the claim that there is a single distinguished truth predicate. Instead, we have a whole family of truth predicates, $T_1, T_2, T_3 \ldots$ *ad infinitum*. The subscript of each predicate indicates its level. We then assign a level to each formula of the language. A formula is level 0 if it contains no truth predicates. It is level 1 if the highest level truth predicate it contains is T_1. It is level 2 if the highest level truth predicate it contains is T_2, and so on. We state the level of a formula by subscripts: A_α indicates A is a formula of level α. The idea is that truth

[21] The factive version of T-I is: the rule: $\frac{A}{A < T\langle A \rangle}$.

predicates only apply to formulas on a lower level. So this corresponds to Tarski's hierarchy of truth.

In this setting we can formulate a workable truth introduction principle. The idea is that truth is grounded in being *at lower levels*. In other words, for every α and positive β we have the following rule schema:

$$\mathrm{T}_k\text{-I}\ \frac{}{\mathrm{A}_\alpha < \mathrm{T}_{\alpha+\beta}\langle \mathrm{A}_\alpha \rangle}$$

How does this solve our first puzzle? It renders the offending sentences unprovable. Specifically, $\exists x T_1 x < T_1\langle \exists x T_1 x\rangle$ is not an instance of this rule. That's because $\exists x T_1 x$ is itself a formula of level 1. The closest substitution instance of this rule is $\exists x T_1 x < T_2\langle \exists x T_1 x\rangle$. But $\mathrm{T}_2\langle \exists x T_1 x\rangle$ is not an instance of $\exists x T_1 x$. So this leads to no irreflexivity violation. So we've solved the problem involving truth. Similar solutions extend to knowledge: perhaps this too has a typed structure. Meanwhile, one can appeal to ramified type theory to solve the problems with higher-order quantification. So Korbmacher's approach seems to have general applicability.

For those who like typed theories of (*inter alia*) truth, this is great. I encourage such people to consider this solution. But many people do not like typed theories of truth. For a start, such theories tends to be technically formidable.[22] One might prefer a simpler solution to these paradoxes. What's more, the syntactic restrictions such theories usually involve seem very odd. Let's update the example from Kripke (1975, 695–96). Suppose Mueller says "Everything Trump says is false." Suppose Trump says "Everything Mueller says is false." Then Mueller's assertion should include Trump's in its scope, and vice versa. But, if truth predicates can only apply to formulas of lower levels, this is impossible. It's easy to multiply examples like this. So it seems typed truth is not a good translation of the English notion of truth. This solution does not solve the English language version of the puzzles. It solves a puzzle for a property a bit like truth, but not for the real McCoy.

Finally, let's consider rejecting irreflexivity and thus asymmetry. If you don't think grounding is irreflexive, then there's no puzzle.[23] But it's not satisfactory to drop irreflexivity without leaving anything in its place. After all, the sorts of irreflexivity failures these puzzles create seem rare. Irreflexivity seems to usually hold. So what we need is some systematic way of restricting irreflexivity. And – ideally– we want some explanation of why we should restrict it in that way. Without such an explanation we've no idea why irreflexivity holds in certain cases and not others.

Woods (2017) has provided such a way. He thinks that the puzzle cases involve what he calls *vacuous* grounding. Consider a grounding fact, (Δ grounds D). A occurs *vacuously* in (Δ grounds D) if one could replace every occurrence of A in Δ for any fact B, and get a true statement of ground out the other end. For example, consider the fact that $\exists x Tx < \exists x Tx$. Any fact could replace $\exists x Tx$ on the left of the

[22] It certainly is in Korbmacher (2018b).

[23] Correia (2014) and Rodriguez-Pereyra (2015) both suggest this. Jenkins (2011) and Barnes (2018) suggest irreflexivity fails for different reasons.

grounding operator. This is because every fact grounds $\exists xTx$. So $\exists xTx$ occurs vacuously in $(\exists xTx < \exists xTx)$. Or consider the fact $\exists x(Tx \wedge Q), Q < \exists x(Tx \wedge Q)$, where Q is any old formula. Suppose you replace the left-hand side occurrence of $\exists x(Tx \wedge Q)$ with any fact whatsoever. Woods thinks you still get out a true statement of ground. This is because, when combined with Q, every fact grounds $\exists x(Tx \wedge Q)$. So this is another case of vacuous grounding. Woods suggests that irreflexivity holds only for cases of non-vacuous grounding. This means that $A, \Delta < A$ can be true iff one could replace all occurrences of A in A, Δ for any fact whatsoever, and still have a true statement of ground.

This is a nice solution to the puzzles. It gives us a systematic way to restrict irreflexivity. So it's my second favourite. But, for two reasons, it leaves me unsatisfied. First, I don't think it solves all our puzzles. In particular, it doesn't solve the puzzles in which we attribute truth to sentences. Consider the truthteller. You can't swap A out for any B in the fact A strictly grounds T'A'. The fact that snow is white does not ground the fact that 'this sentence is true' is true.[24] By the same token, Woods' solution fails to solve the paradoxes of conjunction or disjunction. So it's insufficiently general. Second, is that it lacks explanatory depth. It gives us a systematic restriction of irreflexivity. But it doesn't give us an adequate explanation of why that restriction holds. Why is vacuous grounding consistent with irreflexivity violations and not non-vacuous grounding? What is it about vacuity which is so important? Woods does say something about this. He says that when A occurs vacuously in some grounding fact, C < D, the particular content of A doesn't explain D. But, he thinks, when A occurs non-vacuously in such a grounding fact, the particular content of A does explain D. He suggests there is something problematic about the particular content of A explaining itself. But not about A explaining itself (Woods 2017, 8–9). I don't find this obvious. It is unclear to me why there'd be a problem with the *particular content* of A explaining A, but no problem whatsoever with A explaining A.[25] So, in my eyes, Wood's strategy fails to fully *explain* why irreflexivity fails in cases of vacuous ground. But we're looking for an explanation. So I don't think we have a fully satisfactory solution to the puzzles. In the next section, I'll present my solution. I think it evades the problems these solutions suffer.

7 Solving the puzzles

The solution I'll present rests on the notion of weak ground. Here's how the solution will work: I first put forward some rules for weak ground. I then derive from these some rules for strict ground. The rules for strict ground will be weaker than the rules

[24] Woods recognizes the issue in a footnote (Woods 2017, n. 38).

[25] There is a general question of what it is for the particular content of A to explain something. I've made a few suggestions to Woods. The one he liked best was contrastive. On this view, the particular content of A explains B iff there is some C such that A *rather than C* explains B. This is a likeable suggestion, because in the puzzle cases there is no such C. Yet it remains non-obvious to me why we should balk at there being some C such that A rather than C explains A, but not balk at A explaining A.

we've considered above. But their weakness isn't an *ad hoc* response to these puzzles. It's natural if we're deriving the logic of strict ground from that of weak ground. And that itself is natural if we take weak ground to be more fundamental than strict ground. But this strategy requires a good understanding of weak ground. So, first, we better clarify this notion.

7.1 What is weak ground?

In this section, I want to convey the notion of weak ground. We've already said it's a reflexive grounding relation. And we characterized strict ground in terms of weak ground. We said that A strictly grounds B iff A weakly grounds B and B does not even help to weakly ground A. But we haven't yet given a fully worked-out characterization of weak ground itself. In this section, I'll provide two such characterizations. I've got a favourite. But I suspect the favourite will be controversial. So I offer two because I think my solution works on both ways of understanding weak full ground. In any case, these characterizations should suffice to convey the notion of weak ground.

Let's start with my favourite characterization. On this understanding we see a weak full ground as a disjunct of what gets grounded. In other words, we say that A weakly fully grounds B iff there's some p such that B just is $(p \vee A)$. More generally, let $\hat{\Delta}$ be the conjunction of the sentences in Δ. Then we say that Δ weakly fully grounds B iff there's some p such that B just is $(p \vee \hat{\Delta})$. This means we understand weak partial ground as a conjunct of a disjunct of what gets grounds. A weakly partially grounds B iff there's some p, q such that B just is $(p \vee (q \wedge A))$. On this characterization, weak ground is understood in terms of 'just-is' claims and boolean operators. Weak grounds are, in a sense, parts of what they ground.

What's do these 'just-is' claims express? They express an identity-like connection between the things expressed by sentences.[26] This notion is at work in claims like 'for water to be wet just is for H_2O to be wet' and 'for there to be bachelors just is for there to be unmarried men.' These claims identify 'water is wet' with 'H_2O is wet' and 'there are bachelors' with 'there are unmarried men.' Like identity, this notion is transitive, reflexive and asymmetric. And like identity, it obeys a version of Leibniz's law: if A just is B, then one can substitute As for Bs in certain formula without changing the truth value of those formulas. The formula are those in which neither A nor B occurs in an opaque context. The paradigm examples of opaque contexts are belief contexts. When a formula ascribes a belief in A, sometimes substituting in something identical to A does change the truth value of the formula. Leibniz's law says that, in other contexts, you can substitute identicals salva veritate.

Why might this characterization of weak full ground be controversial? To explain that, we'll have to distinguish between two notions of ground: worldly and representational ground. The best way to distinguish between these notions, I think, invokes the above notion of identity. Representational ground induces opaque

[26] See Dorr (2016) for an extensive discussion.

contexts. Worldly ground does not.[27] So if A just is B, then in statements of worldly ground one can substitute As for Bs salva veritate. But this isn't guaranteed in statements of representational ground. So it might be that A representationally grounds B even though A just is B. But it can never be that A worldly grounds B even though A just is B.

The characterization might be controversial because it was originally offered for just worldly ground.[28] So, it's unclear whether it can be extended to representational weak full ground. But fortunately I think it can. To extend it, we do have to make another distinction. This is a distinction between two kinds of identity: worldly and representational identity. Paradigm cases of worldly identity are given above. Representational identity is a more fine-grained notion. Paradigm cases of representational identity include commutativity: $(A \vee B)$ just is $(B \vee A)$. To gloss this distinction more generally, we can help ourselves to a distinction between the way a statement represents the world as being and how the statement represents the world as being that way.[29] Worldly identity is only sensitive to the way a statement represents the world as being. Representational identity is sensitive to how that statement represents the world as being that way. With this distinction in hand, we can extend our characterization to representational weak full ground. We say that A representationally weakly fully grounds B iff there's some p such that B just is $(p \vee A)$. But we interpret the 'just-is' claim as expressing representational identity, not worldly identity. So we can think of both notions of ground as disjuncts of what gets grounded.

But this itself may be controversial.[30] So it's worth looking at an alternative way to spell out weak full ground. The alternative I'll look at involves explanatory subsumption. At a rough approximation, we say that A weakly grounds B iff A explains everything B explains. We can understand the relevant explanatory relations as relations of strict ground. So this says that A weakly fully grounds B iff A strictly grounds everything B strictly grounds. But, as deRosset (2013b, 16–17) makes clear, this is at best a rough approximation. Problems arise with claims which don't strictly ground anything. *Everything is true* is a good example of such a claim. Since such claims don't strictly ground anything, everything strictly grounds what they strictly ground. Explanatory subsumption is satisfied vacuously. So this

[27] This is from Correia (2017b, 58) and Correia and Skiles (2017, 15).

[28] See Correia and Skiles (2017, 19).

[29] See Fine (2017, 685–86).

[30] I can think of some *bad* reasons to reject it. Here's one: A weakly fully grounds A but then this requires $\exists p(A$ just is $A \vee p)$. And there's no such p. So you can't guarantee everything weakly grounds itself. Here's why this is a bad reason: I think A just is $A \vee A$. So A itself is such a p. So we can guarantee that $A \leq A$. Of course it's controversial that A just is $A \vee B$ in the representational sense of 'just is.' This isn't part of the system in Correia (2017a). But the rules I'll present in the next section are inconsistent with this system anyway. So it's hardly worrying that this claim conflicts with those rules. Here's another bad reason: this characterization clashes with the strict ground principles we began with. For example, $A \vee A$ just is $(A \vee (A \vee A))$. So $(A \vee A)$ weakly fully grounds A. But this conflicts with $\vee$-I. This is bad for much the same reason. In the next section I'll say we *should* restrict rules like $\vee$-I. So again it's not worrying that our characterization conflicts with such rules.

approximation implies that everything weakly fully grounds such claims. But that simply isn't the case. The fact that it's raining is no full ground, in any sense, of everything is true.

So we need to be a bit more careful. deRosset himself points out that we can evade this by stipulating the generalization must be satisfied non-vacuously. Then nothing weakly fully grounds *everything is true*. The problem with this is that it implies *everything is true* doesn't weakly ground itself. But, on the intended notion of weak ground, everything weakly grounds itself. Yet there's a reasonably straightforward solution to this problem. We invoke both non-vacuous generalization and identity. We can say that A weakly grounds B iff B strictly grounds something and A strictly grounds everything B strictly grounds, *or* A just is B. This second clause ensures that even explanatory freeloaders weakly ground themselves. So this version of the explanatory subsumption characterization escapes deRosset's criticisms.

This means we have two workable characterizations of weak ground. Before moving on, I want to do two thing. First, I want to clarify the nature of these characterizations. Above, these characterizations are formulated as material biconditionals. They tell us what weak ground is materially equivalent to. But a more illuminating interpretation of them –one I endorse– is as 'just-is' claims. They don't just tell us what statements of weak ground are equivalent to; they tell us what they're identical to. They tell us what weak ground is. So construed, it's hard to see what more could be done to convey the concept of weak ground. Second, I want to address a worry about the explanatory subsumption characterization. The worry is that, in this case, weak ground is characterized partly in terms of strict ground. But, in Sect. 2, strict ground was characterized partly in terms of weak ground. One might worry that this circularity impairs the ability of this characterization to convey the notion of weak ground. But it seems to me that it does not. Here's an analogous case: we could convey the notion of conjunction by saying that $A \wedge B$ just is $\neg(\neg A \vee \neg B)$. The thought that $A \vee B$ just is $\neg(\neg A \wedge \neg B)$ wouldn't imperil this. So this sort of circularity doesn't seem problematic for my purposes. More generally, the situation on this second characterization is that strict ground and weak ground are inter-definable, much like conjunction and disjunction are inter-definable. But inter-definability is not a problematic type of circularity. So both of the above characterizations of weak ground seem workable. Weak ground will be the workhorse notion for the solution to the puzzles I'm about to present. Let's now move to that solution.

7.2 The logic of weak ground

The strategy is to derive the logic of strict ground from that of weak ground. We first add weak full ground ($\leq$) and weak partial ground ($\preceq$) to the vocabulary of our language. We then add the following two clauses to our grammar:

- If A and B are wffs, then $(A \preceq B)$ is a wff
- If Δ is a list and B is a wff, then $(\Delta \leq B)$ is a wff

This puts us in a position to articulate a system of weak ground. The driving idea is to take the weak ground counterparts of all the principles in the previous section. That is, we accept those principles are plausible for *some* notion of ground. But we take this to be the weak rather than the strict notion. So the principles we'll have are:

$$\exists\text{-I}(\leq)\ \frac{}{A \leq \exists A^{[t/v]}} \qquad \exists\text{-I}^{h}(\leq)\ \frac{}{A \leq \exists V A^{[B/V]}}$$

$$\forall\text{-I}(\leq)\ \frac{}{A \leq \forall v A^{[t/v]}}$$

$$\vee\text{-I}(\leq)_1\ \frac{}{A \leq A \vee B} \qquad \vee\text{-I}(\leq)_2\ \frac{}{B \leq A \vee B}$$

$$\wedge\text{-I}(\leq)\ \frac{}{A, B \leq A \wedge B}$$

$$\text{T-I}(\leq)\ \frac{}{A \leq T\langle \text{A} \rangle} \qquad \text{K-I}(\preceq)\ \frac{}{A \preceq Ks\langle \text{A} \rangle}$$

$$\text{T-I}^{s}\ \frac{\text{`A' expresses A}}{\text{A} \preceq \text{T `A'}}$$

These are like the principles from the previous section.[31] But we've swapped strict ground for weak ground. This means you'll be able to derive symmetric cases of weak ground from these rules. But that's no problem: weak ground isn't meant to be asymmetric. So these rules alone won't lead to any contradictions. Changing connections of strict ground into weak ground gives us non-paradoxical rules.

But this by itself is not a satisfactory solution. It is not enough to just say that all the connections of strict ground are connections of weak ground. That's because we want a system which will generate the plausible instances of the strict ground rules. Otherwise we're doing what I objected to before. We're rejecting these rules without putting anything adequate in their place. So we need a way to generate rules for strict ground. Here's how to do this. We just define strict ground in terms of weak ground. Let $(A \npreceq B)$ abbreviate $\neg(A \preceq B)$. Our definitions are:[32]

[31] This is essentially an extension of the system presented in Lovett (2019).

[32] These definitions are formalization of those is Fine (2012, 51–52).

$$\text{Def}(\prec)\ \frac{}{A \prec B \leftrightarrow A \preceq B \wedge B \npreceq A}$$

$$\text{Def}(<)\ \frac{}{A_1, A_2 \ldots < B \leftrightarrow ((A_1, A_2 \ldots \leq B) \wedge ((B \npreceq A_1) \wedge (B \npreceq A_2))) \ldots}$$

With these definitions in hand it's easy to derive a gamut of rules for strict ground. Here they are:

$$\exists\text{-I}(<)\ \frac{(\exists v A^{[t/v]}) \npreceq A}{A < (\exists v A^{[t/v]})} \qquad \exists\text{-I}^h(<)\ \frac{(\exists V(A^{[B/V]})) \npreceq A}{A < (\exists V(A^{[B/V]}))}$$

$$\forall\text{-I}(<)\ \frac{(\forall v A^{[t/v]}) \npreceq A}{A \prec (\forall v A^{[t/v]})}$$

$$\vee\text{-I}(<)_1\ \frac{A \vee B \npreceq A}{A < A \vee B} \qquad \vee\text{-I}(<)_2\ \frac{A \vee B \npreceq B}{B < A \vee B}$$

$$\wedge\text{-I}(<)\ \frac{A \wedge B \npreceq B \qquad A \wedge B \npreceq A}{A, B < A \wedge B}$$

$$\text{T-I}(<)\ \frac{T\langle \text{A} \rangle \npreceq A}{A < T\langle \text{A} \rangle} \qquad \text{K-I}(<)\ \frac{Ks\langle A \rangle \npreceq A}{A \prec Ks\langle \text{A} \rangle}$$

$$\text{T-I}^s\ \frac{\text{‘A’ expresses A} \qquad \text{T ‘A’} \npreceq \text{A}}{\text{A} \prec \text{T ‘A’}}$$

We derive these rules by taking the rules for weak ground and then applying the definitions. They're restrictions on the rules for weak ground. When the condition above the line is met, you can derive a statement of strict ground. But the important thing is that these rules evade paradox. To show that we need just one further principle:

$$\text{Subsumption}(\leq/\preceq)\ \frac{\Delta, A \leq B}{A \preceq B}$$

This just says you can weaken relations of weak full ground to those of weak partial ground. With this in hand consider the paradox we started with. Let S be the fact that something is true. By E-I($\leq$), T$\langle$S$\rangle$ is a weak full ground of S. By T-I($\leq$), S is a weak full ground of T$\langle$S$\rangle$. But then by Subsumption($\leq/\preceq$) each weakly partially grounds the other. So we can't apply either T-I(<) or E-I(<) to generate an asymmetry violation. So, no paradox. Now consider the knowledge-based puzzle.

By E-I($\leq$), the fact that Adam knows ⟨Someone knows something⟩ is a weak full ground of the fact that someone knows something. So it's a weak partial ground of this fact. And the fact that someone knows something is, by K-I($\preceq$), a weak partial ground of the fact that Adam knows ⟨Someone knows something⟩. So each weakly partially grounds the other.[33] So we can't apply either T-I(<) or K-I(<) to generate an asymmetry violation. So, no paradox. Similar remarks apply to the other paradoxes. We have a system which evades the puzzles of ground.

But this might still not yet seem like a satisfactory solution to the puzzles. This is because, although these rules don't generate paradox, it's not exactly clear what they do generate. Just when are the restrictions satisfied? When, for instance, is it that $A \vee B \npreceq A$ or that $\exists xFx \npreceq Fa$? There are two reasons why this is important. First, we need some way of showing these rules generate the plausible instance of the former rules. Otherwise they're no real advance. Second, strength is itself a theoretical virtue. Theories which tell us more about the world are better. If these restricted rules allow us to generate very few connections of strict ground, then that's a vice. We'll have a solution to the puzzles of ground but at the cost of accepting a much worse system.

I think both points are most pressing for the rules dealing with logical operators. It's for these that strength is most important and it's these that have the most intuitively plausible instances. Fortunately –especially in these cases– we can identify some non-trivial conditions sufficient for meeting the restrictions. To do this, we'll need to rely on the transitivity of weak partial ground. That is:

$$\text{Trans } (\preceq/\preceq)\ \frac{A \preceq B \qquad B \preceq C}{A \preceq C}$$

This shouldn't cause any alarm. It seems essential to any workable logic of ground. So let's start with disjunction. When can we be sure that $A \vee B \npreceq A$? This will tell us when A strictly grounds $A \vee B$. It's enough for this that B does not weakly partially ground A. Here's the argument: by $\vee$-I$(\leq)_1$, B weakly fully grounds $A \vee B$. So, by Trans ($\preceq/\preceq$) and Subsumption($\leq/\preceq$), if $A \vee B$ weakly partially grounds A, then B also weakly partially grounds A. So if B doesn't weakly partially ground A, then nor can $A \vee B$. Mutatis mutandis, the same goes for the

[33] Correia (2017a, 525) argues that weak partial ground is antisymmetric. This would make this conclusion untenable. But he characterizes weak full grounding quite differently to how it's characterized Sect. 7.1 (Correia 2017a, 516). He defines weak full ground as strict ground or identity. In the singular case, he says $A \leq B$ iff $A < B$ or A just is B. From such a definition it follows that weak partial ground is antisymmetric. So this is not one of the characterizations of weak full ground on which the solution in the text works. But on the characterizations I give in Sect. 7.1 it seems to me implausible that weak partial ground is antisymmetric. Consider the disjunction characterization. On this view A weakly partially grounds B iff there's some p, q such that A just is $(p \vee (q \wedge B))$. B weakly partially grounds A iff there's some p_1, q_1 such that $(p_1 \vee (q_1 \wedge A))$. But this doesn't even guarantee that A and B are materially equivalent. So it can hardly be thought to guarantee that A and B are identical. Similar remarks go for the explanatory subsumption characterization: the fact that A *helps* explain everything B helps explain and vice versa hardly seem to guarantee that they're identical. So we needn't think weak partial ground is antisymmetric. That's not to say there isn't an antisymmetric notion in the vicinity. Following Correia and Skiles (2017, 19), I suspect that weak *full* ground is antisymmetric. But this doesn't cause any problems for the proposed solution. Thanks to a helpful referee for raising this point.

other disjunction rule. Now consider conjunction. When can we be sure that $A \wedge B \npreceq A$ and that $A \wedge B \npreceq B$? It's enough that neither A nor B weakly partially ground the other. The argument is similar. By $\wedge$-I($\leq$), $A, B \leq A \wedge B$. So, if $A \wedge B \preceq B$, then $A \preceq B$. So if $A \npreceq B$, then $A \wedge B \npreceq B$. A parallel argument shows that $B \npreceq A$ implies $A \wedge B \npreceq A$. So the restrictions for disjunction and conjunction are met when the disjuncts and conjuncts don't weakly partially ground one another. Under these conditions we can derive connections of strict ground.

What about the quantifiers? For both universal and existential quantification, the restriction is met when $\exists v(A^{[t/v]} \npreceq A)$. In other words, suppose there's some v such that $A^{[t/v]}$ doesn't weakly partially ground A. So A is not grounded by *every* instance of $\exists v A^{[t/v]}$. Then the restriction is met. I'll just show this for atomic predications. Suppose $\exists x(Fx \npreceq Fa)$. Then apply existential elimination to get $Fc \npreceq Fa$. Now by $\exists$-I($\leq$) it follows that Fc weakly fully grounds $\exists x Fx$. So, given transitivity, it can't be that $\exists x Fx$ weakly partially grounds Fa. A similar argument applies to all the other quantifiers. So we can derive connections of strict ground if (some) instances of the quantifications do not stand in certain ground-theoretic relationships. We need just one instance of $\exists v A^{[t/v]}$ to not ground A.

This has two implications. First, it means that we generate the plausible instances of the logical rules for strict ground. Consider the relationship between my jacket's redness and the fact that something is red. There are some things the redness of which doesn't even weakly ground my jacket's redness. Mars' redness doesn't even weakly ground my jacket's redness. So we can apply $\exists$-I($<$) to derive that my jacket's redness strictly fully grounds the fact that something is red. Second, it means adopting these restricted rules doesn't cripple the strength of the system. This is starkest in the case of the quantifiers. I think it is *very* rare that every x is such that Fx weakly ground Fa. So these rules give us plenty of strict grounds for quantifications. But this also goes for the truth functors. It's pretty rare that the disjuncts of disjunctions ground one another. So these principles will –given this background knowledge– generate a lot of strict grounding connections.

We do need the background knowledge. We need to know that certain facts don't weakly partially ground other facts. But we can generate this from our characterizations of weak ground. Consider the explanatory subsumption characterization. Mars' redness doesn't help subsume the explanatory role of my jacket's redness. The latter explains all kinds of things the former does not. It explains why my jacket is red or blue, why my jacket is coloured, why my jacket matches my shoes and so on. And they're certainly not identical. So the former must not weakly ground the latter. Second, consider the disjunction characterization. Mars' redness is not a conjunct of a disjunct of my jacket's redness. There's no p, q such that for Mars to be red just is for either p or (q and my jacket is red). So again the former must not weakly ground the latter. So these characterizations help buttress our knowledge of ground-theoretic isolation.

That deals with the logical rules. The story is a bit more mixed when it comes to the predicational rules. In the next section I'll suggest that for propositional truth the restriction is never met. What we have here is really identity rather than strict ground. But that's getting ahead of ourselves. We can say something less radical in the case of knowledge. My knowledge that it's raining does not ground the fact that it's raining. Not even partially. Our characterizations of weak ground establish as much. So we get the intuitively plausible case(s) of knowledge grounding. The fact it's raining is a strict partial ground of my knowledge thereof. Mutatis mutandis, the same remarks go for the attribution of truth to sentences. The truth of 'it's raining' doesn't even weakly partially ground the fact that it's raining. So it seems to me we have what we need when it comes to these rules. In all these cases we generate the intuitively plausible principles. And we do so without making the system unconscionably weak.

Let's sum up. We needed a way to systematically restrict the strict ground principles. I've presented a way. It involves deriving these principles from comparable principles of weak ground. One does this by defining strict ground in terms of weak ground. The weak ground principles don't generate paradox. That's because weak ground isn't asymmetric. The derived principles don't generate paradox. That's because they're restricted. So we've got a solution to the paradoxes. And it seems to me a satisfactory solution. That's because these restricted principles make up a satisfactory system of strict ground. They generate paradigm cases of strict ground. And they tell us a lot about what strictly grounds what. So we've restricted the paradoxical rules without mauling the logic of ground.[34]

8 Motivating a restriction

So we have a solution to the puzzles. But so far the solution is mainly technical. I've explained how systematically restricting the strict grounding principles can avoid paradox. And I've argued that the system you end up with is adequate. But I haven't provided a deep motivation, independent of the paradoxes, for restricting these principles. I have provided some motivation. The restricted principles are those you get if you derive the logic of strict ground from that of weak ground. And I've said why you might want do that. You might think weak ground is the more fundamental notion. But it'd be nice if there was some further motivation for restricting the strict ground principles. The restrictions I've just offered would then seem a promising way to meet an independent need to restrict these principles.

There is such a motivation. It swings on particular views of the grain of reality. On these views, reality is relatively coarse-grained. We spell this out via the identity-like connections we mentioned in Sect. 7.1. These were the connection expressed in 'just is' claims. When we say 'for Cicero to be eloquent just is for Tully to be eloquent' or 'for there to be squares just is for there to be rectangular equilaterals' we are expressing these identities. More coarse-grained views of reality accept more of these identities. More fine-grained views accept fewer. In the rest of this section we'll see how various coarse-grained views provide independent motivations for restricting the strict ground principles.

[34] To get a satisfactory logic we do of course need to add some ancillary rules. We need at least those from Fine's pure logic (Fine 2009). I omit these because they aren't necessary for generating the puzzles.

The first such view concerns what logic adds to reality. On this view, the answer is: not much. Adding logical operators to a fact, without changing its truth value or adding content, doesn't create a new fact. Let '$\equiv$' stand for the relevant notion of identity. Then if you accept this view, you accept (*inter alia*) the following:

$$\text{(i)} \quad A \equiv A \vee A \qquad \text{(ii)} \quad A \equiv A \wedge A$$
$$\text{(iii)} \quad \exists vA \equiv \exists v \exists vA \qquad \text{(iv)} \quad \exists VA \equiv \exists V \exists VA$$
$$\text{(v)} \quad \forall vA \equiv \forall v \forall vA$$

If you accept these identities, then you must restrict the strict ground claims we began with. At least that's the case if your notion of ground obeys Leibniz's law. Remember Leibniz's law implies that if A just is C, and A grounds B, then C grounds B. So consider, for instance, $\vee$-I. With this we can infer $A < A \vee A$. But by (i) $A \equiv A \vee A$. So, by Leibniz's law, it follows that $A < A$. Or consider $\exists$-I. With this we can infer that $\exists xFx < \exists x \exists xFx$. But by (iii), $\exists xFx \equiv \exists x \exists xFx$. So by Leibniz's law, we can infer that $\exists xFx < \exists xFx$. Similar points go in the other cases. So these identities provide independent motivation to restrict some of the strict ground principles.

A second view goes a bit further. Suppose it's not just logical operations which fail to make for a genuine distinction. We might think truth ascriptions add nothing to reality. That is, one might think there's no difference between P and the claim that $\langle P \rangle$ is true. If you accept this view, then you accept:

$$\text{(vi)} \quad A \equiv T\langle A \rangle$$

This schema is in line with the identity theory of truth. And this is one way, although not the only way, to articulate a deflationary theory of truth.[35] But it renders T-I untenable. For suppose A strictly grounds T$\langle A \rangle$. Then since $A \equiv T\langle A \rangle$, by Leibniz's law it would follow that A strictly grounds itself. This problem is distinct from the puzzles with which we began. So it generates an independent motivation for restricting T-I.

The situation is a little more complex with the T-I^s and K-I rules. Let's begin with T-I^s. Suppose one accepts the restricted versions of the other rules. Then there seems to me little reason to accept the unrestricted version of this rule. For consider why one might accept this unrestricted version. One reason is by analogy with the truth-introduction rule for propositions. If we've restricted this, the analogy fails. A second reason goes via an identity. Suppose we think T'A' $\equiv \exists P\,(P \wedge$ ('A' expresses P)). This says that for 'A' to be true just is for there to be some fact, p such that p is the case and 'A' expresses p. And suppose we accepted $\exists$-I^h. Then we could derive T-I^s. But once we've restricted $\exists$-I^h that derivation fails. We can only derive the restricted version of T-I^s. So the prior restrictions *undercut* our reason to accept this unrestricted T-I^s rule.

[35] Some people who like deflationary theory of truth deny there are such things as propositions. They would presumably prefer to articulate the deflationary theory some other way. Thanks to a referee for this point.

What about K-I? Here I think we can say something similar. The appeal of K-I may also rest on the analogy with truth. But once we've restricted the truth rules, then this analogy fails. Or the appeal may rest in part on an identity. Namely, let *Bsp* stand for S believes P and *Wsp* stand for S has warrant for P. Here 'warrant' means justification and anti-Gettier conditions. The relevant identity is: $Ksp \equiv P \wedge Bsp \wedge Wsp$. This says, roughly, that for S to know P just is for S to have a true warranted belief in P. With this and $\wedge$-I, one could derive that P strictly partially grounds *Ksp*. But once one has restricted $\wedge$-I this derivation no longer goes through. So this also undercuts our reason to accept the K-I rule.

Whether these rules have other sources of appeal I am unsure. But, in both cases, we can also add one more point. We can make an appeal to theoretical uniformity. Specifically, all these other strict grounding principles are now restricted. You can apply none of them unless you know certain weak partial grounding relations don't hold. So it looks a little odd that these principles would not be so restricted. This counts as some pressure –pressure from theoretical unity– to restrict these principles. And, at the least, it means it no longer looks *surprising* that they are restricted. So are all the others. So coarse-grained views of reality provide independent motivation for restricting the strict ground principles. On such views, we end up with a weaker logic of strict ground because we have a stronger logic of identity. Restricted principles of strict ground are mandated by unrestricted principles of identity.

Now, this motivation is not completely general. It's difficult to use it to motivate restricting strict ground rules for a representational ground. This is because the case for these identities is strongest when they're construed as worldly identities. And it could be that A just is $T\langle A\rangle$, in the worldly sense, but that A representationally grounds $T\langle A\rangle$. So this just motivates restricting the strict ground rules for worldly ground. I want to close with two comments about this. First, this doesn't mean the solution in Sect. 7 works only for worldly ground. It just means that, when it come representational ground, you can't *motivate* this solution in the way just sketched. But you can still accept the system I've proposed. You just don't have a motivation for accepting the system *independent* of solving the puzzles. That makes the solution less satisfying. But one might think that solving the puzzles is motivation enough.

Second, there's a question about how important this limitation is. I do not think it is so important. To see why, it'll help to say why we should worry about these puzzles of ground. We should worry about them, I think, because they threaten a notion we need for certain tasks. These are the tasks noted in Sect. 1. We need ground to make sense of certain paradigm cases and to play certain roles in metaphysics. The worry is that, if the notion of ground can't escape paradox, we don't have a notion which can fulfil these tasks. But it seems to me that worldly ground alone will do these jobs. It holds in the paradigm cases. The connection between sets and their members, for instance, is a connection of worldly ground. And it can play the theoretical roles. We can, for example, formulate key metaphysical theories in terms of worldly ground. So, in order to fulfil these purposes, it suffices to ensure worldly ground is non-paradoxical. And that is what our motivation does. So, even were my solution to the puzzles to only apply to worldly ground, this wouldn't be a very important limitation. We could still discharge the tasks which motivated introducing the notion of ground in the first place.[36]

9 Conclusion

We began with a puzzle. The puzzle imperilled a distinctive type of non-causal explanation: ground. It showed that *prima facie* plausible principles about this notion were inconsistent. I've provided a solution to this puzzle. The solution involves restricting those principles. To do this we derived claims of strict ground from those of weak ground. This gave us a quite comprehensive set of grounding rules. I think this solution has three main virtues. First, it has general applicability. It applies to all the puzzles I've discussed in this paper. Second, the restricted principles inherit the plausibility of the unrestricted ones. That is because the plausibility of the unrestricted rules rests on the fact that they generate paradigm cases of ground. But the restricted principles also generates these paradigm cases. Third, restricting the principles can be independently motivated. Accepting a coarse-grained view of reality forces us into some restrictions anyway. So the solution we've seen seems to me promising. In fact, I know of no other solution with these virtues.

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[36] There might be other tasks for which we need representational ground. Correia (2017b), for example, suggests representational ground can help give us an account of real definition. But it's not clear that this makes representational ground indispensable. That's because Correia thinks we can formulate his account of real definition in terms of another notion: comparative joint-carvingness. Now he does also argue that comparative joint-carvingness and representational ground are equivalent (Correia 2017b, 65–70). But, if representational ground is paradoxical, then this seem to give us good motivation to resist these arguments.

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